INTERFERENCE DIFFRACTION

Conditions necessary for steady Interference Pattern

>> *The two sources of light should be coherent* : Coherent means having a constant phase difference between them at all times. Best achievable by deriving the two sources from a single source

>>*The two sources must be monochromatic* : The position as well as width of the fringes depends on wavelength of light. Hence fringes with different colours will not be coinciding.

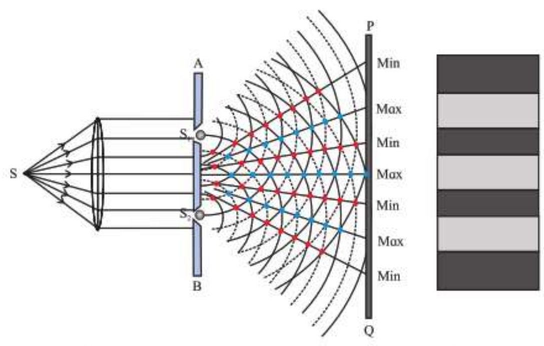
>> *The two interfering wave must have same amplitude :* Only then will the dark be of zero intensity

>> *The separation between the two slits must be small in comparison to the distance between the slits and the screen*

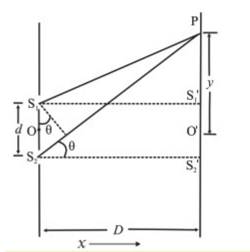
>> *The two slits should be narrow :* else will cause blurring

>> *The two waves should be in the same state of polarization*

Youngs Double Slit Experiment:



S1 and S2 are two coherent source (same frequency and constant phase difference at all times) equidistant from S. Where crest meets crest or trough meets trough and we observe high intensity bright fringe. Where a crest of one coincides with trough of the other, we get dark fringe on the screen PQ. We get alternate dark and bright fringes called interference pattern



Analytical Study

d: distance between the slits

D: distance between slit and screen

At O' distance travelled by the secondary wavelets from S1 and S2 is equal, hence O' is a central bright fringe.

Now consider a point P on the screen.

$(S_2P)^2 = D^2 + (y + d/2)^2$

$(S_1P)^2 = D^2 + (y - d/2)^2$

Thus $(S_2P)^2 - (S_1P)^2 = 2yd$

Therefore, $(S_2P - S_1P)(S_2P + S_1P) = 2yd$

since d<<D, we assume $S_2P = S_1P \approx D$

Thus, path difference $\Delta p = S_2P - S_1P = 2yd / 2D = yd/D$

For constructive interference, $\Delta p = n\lambda$

$yd/D = n\lambda$, Thus $y = n\lambda D/d$ for n=0,1,2,3.....

For destructive interference, $\Delta p = (2n - 1)\lambda/2$

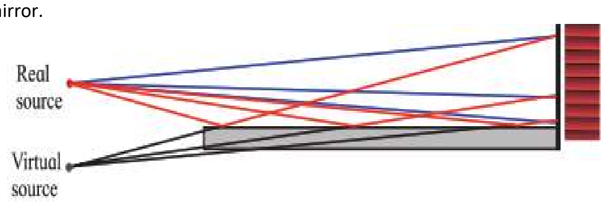
$yd/D = (2n - 1)\lambda/2$, Thus $y = (2n - 1)\lambda D/2d$ for n=1,2,3.....

Fringe Width (W)= distance between two consecutive bright bands or two consecutive darks bands = $\lambda D/d$

Thus, both dark and bright fringes are of equal width and are alternately placed on both sides of the central bright band

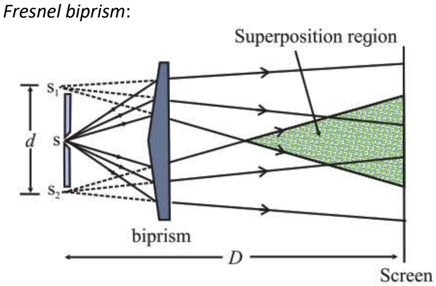
Methods of obtaining the two coherent source:

Lloyd's mirror: A light source is made to fall at a grazing angle on a plane mirror.



Some light falls directly on the screen while some reflects before hitting the screen. The reflected light appears to come from a virtual source. Since it derived from the same source, the two source are coherent.

Fresnel biprism:

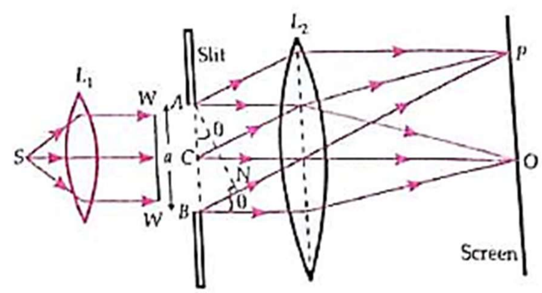


Vertex angle nearly 180°. Very small refracting angle of 30' to 1° with the base. The refracting edge parallel to the slits. The biprism produces S_1 and S_2 as the two virtual sources from the same source S, hence they both are coherent.

Diffraction: Light is seen to bend around edges of obstacles and enter into regions where shadows are expected. This bending of light is called diffraction of light.

Diffraction is noticeable only of the obstacle is of the order of wavelength.

Diffraction at Single Slit (Fraunhofer Diffraction) :



Source S of monochromatic light is placed at the focus of a lens L1 to generate a parallel wavefront WW which is incident on a narrow slit AB of width 'a'. According to Huygens' theory, all parts of this slit becomes a source of secondary wavelets. These spread in all directions, thus causing diffraction. L2 is used as a focusing lens. The screen is places on the focal plane of this lens.

Central or primary maxima: All the secondary wavelets going straight from AB are focused at O, thus producing a central bright fringe.

Width of the central fringe = $2y = 2\frac{\lambda D}{a}$

Half or semi angular width of central fringe = $\theta = \frac{y}{D} = \frac{\lambda}{a}$

Angular width of central maximum = $2\theta = 2\frac{\lambda}{a}$

Secondary minima: path difference p =BP – AP = BN = ABsinθ = asinθ

Let P be a point on the screen where $p = \lambda$ or $a\sin\theta_1 = \lambda$. Here we divide AB into two halves AC and CB and every point in upper half AC, there will be a point in the lower half (a/2 distance away) where path difference will be λ/2. Hence all the wavelets in the two halves will be in opposite phase and will interfere destructively, creating a minima.

$$\frac{a}{2}\sin\theta_1 = \frac{\lambda}{2}. Hence, \boldsymbol{a}\sin\theta_1 = \boldsymbol{\lambda}\ for\ 1st\ minima$$

Similarly for second, third, fourth minima and so on we can assume the slit is divided into 4, 6, 8 parts and so on. Hence,

$$\frac{a}{4}\sin\theta_2 = \frac{\lambda}{2}. Hence, \boldsymbol{a}\sin\theta_2 = \boldsymbol{2\lambda}\ for\ 2nd\ minima$$

$$\frac{a}{6}\sin\theta_3 = \frac{\lambda}{2}. Hence, \boldsymbol{a}\sin\theta_3 = \boldsymbol{3\lambda}\ for\ 3rd\ minima, and\ so\ on$$

$$In\ general, \boldsymbol{a\sin\theta = n\lambda},\ \ \boldsymbol{for\ n = 1, 2, 3 \ldots produces\ minima}$$

$$\sin\theta \approx \theta = \frac{y}{D} = \frac{y}{f} = \frac{n\lambda}{a}$$

$$y = n\frac{\lambda D}{a} = n.FW$$

$$Thus\ dark\ fringes\ are\ obtained\ at\ \frac{\lambda D}{a}, 2\frac{\lambda D}{a}, 3\frac{\lambda D}{a} \dots\dots$$

$$Distance\ between\ two\ consecutive\ dark = FW = \frac{\lambda D}{a}$$

Secondary Maxima: Let P be a point on the screen where p = 3λ/2
Here we divide AB into 3 parts such that wavelets of each section will have corresponding wavelet in the next section having path difference of λ/2. Hence the adjacent sections interfere destructively and one section will be left out without destruction. This will create the maxima.

$$\frac{\boldsymbol{a}}{\boldsymbol{3}}\sin\boldsymbol{\theta}_1' = \frac{\boldsymbol{\lambda}}{\boldsymbol{2}}.Hence, \boldsymbol{a}\sin\boldsymbol{\theta}_1' = \boldsymbol{3}\frac{\boldsymbol{\lambda}}{\boldsymbol{2}}\ for\ 1st\ secondary\ maxima$$

Similarly, for second, third, forth secondary maxima and so on we assume slit is divided into 5, 7, 9 parts and so on. Hence,

$$\frac{\boldsymbol{a}}{\boldsymbol{5}}\sin\boldsymbol{\theta}_2' = \frac{\boldsymbol{\lambda}}{\boldsymbol{2}}.Hence, \boldsymbol{a}\sin\boldsymbol{\theta}_2' = \boldsymbol{5}\frac{\boldsymbol{\lambda}}{\boldsymbol{2}}\ for\ 2nd\ seconday\ maxima$$

$$\frac{\boldsymbol{a}}{\boldsymbol{7}}\sin\boldsymbol{\theta}_3' = \frac{\boldsymbol{\lambda}}{\boldsymbol{2}}.Hence, \boldsymbol{a}\sin\boldsymbol{\theta}_3' = \boldsymbol{7}\frac{\boldsymbol{\lambda}}{\boldsymbol{2}}\ for\ 3rd\ seconday\ maxima$$

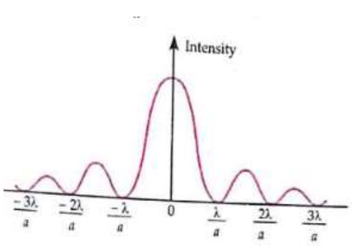
In general, **a sin θ = (2n+1)λ/2 for n=1,2,3,…. Produces secondary maxima**

$$\sin\theta \approx \theta = \frac{y}{D} = \frac{y}{f} = \frac{(2n+1)\lambda}{2a}$$

$$y = (2n+1)\frac{\lambda D}{2a} = (2n+1).\frac{FW}{2} = \left(n+\frac{1}{2}\right)FW$$

$$Thus\ bright\ fringes\ are\ obtained\ at\ 1.5\frac{\lambda D}{a}, 2.5\frac{\lambda D}{a}, 3.5\frac{\lambda D}{a} \dots\dots$$

$$Distance\ between\ two\ consecutive\ bright = FW = \frac{\lambda D}{a}$$

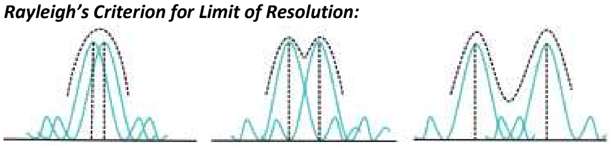


Since only one of the three portions in case of 1st secondary maxima is incident on the screen, and one of the five parts in case of 2nd secondary maxima is incident on the screen and so on, we can justify why there is a significant drop in intensities at each maxima.

Resolving Power: The ability to distinguish two physically separated objects as two distinct objects is known as resolving power of an optical instrument and the minimum visual angle between the two objects required to see two objects as two is called resolving distance or limit of resolution.

$$Resolving\ power = \frac{1}{Resolving\ distance}$$

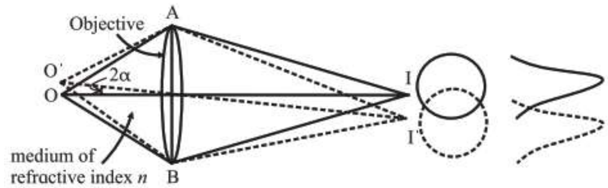
Rayleigh's Criterion for Limit of Resolution:



(a) Unresolved (b) Just resolved (c) well resolved

According to this criterion, the objects are just resolved of the 1st minimum of diffraction of one source coincides with the central maximum of the other.
So for linear objects, distance between the 1st minima and central maxima is λ/a.
Thus, limit of resolution = dθ = λ/a
and minimum separation should be y = λD/a

In case of a microscope,



for non – luminous object

$$\text{Limit of resolution} = \frac{\lambda}{2\mu sin\alpha} = \frac{\lambda}{2\ N.A.}$$

$$\text{Resolving power} = \frac{2\mu sin\alpha}{\lambda} = \frac{2\ N.A.}{\lambda}$$

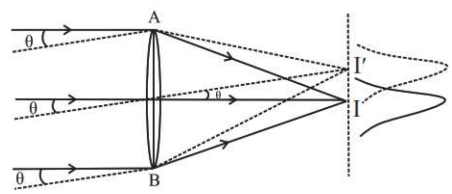
Where μ is the refractive index of the medium (many times the sample is placed in a solution of refractive index μ)
N.A. : Numerical Aperture

For self – luminous object

$$\text{Limit of resolution} = \frac{1.22\,\lambda}{2\mu sin\alpha} = \frac{1.22\,\lambda}{2\ N.A.}$$

$$Resolving\ Power = \frac{2\mu sin\alpha}{1.22\,\lambda} = \frac{2\ N.A.}{1.22\,\lambda}$$

For telescope,



$$\text{Limit of resolution} = \theta = \frac{1.22\,\lambda}{D},\quad where\ D: Aperture\ of\ the\ lens$$

$$Resolving\ Power = \frac{D}{1.22\,\lambda}$$

EXTRA:
OPTICAL PATH:
For a path length d in a medium of refractive index μ, the optical path is given by μ.d, which is the equivalent path in vacuum.
Hence, path difference by introducing the medium is μd – d = (μ-1)d
Say we introduce a transparent plate of thickness t and refractive index μ in front of slit S_1 in interference experiment. Then the optical path of S_1P will now be longer ($\mu.S_1P$). Thus, the entire fringe pattern (including central maximum) will shift.